Advances in Discrete Element Method Application to Grinding Mills

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ABSTRACT: Discrete element method (DEM) has made significant impact in the design and operation of grinding mills. The internal charge motion of grinding balls and ore can be readily examined. As a result, this method enables one to design and examine mill internals via simulation. This manuscript details essential advances made with DEM. These include mill power draft calculations, shell lifter design, lifter wear prediction, pulp lifter simulation and ore particle breakage in grinding mills. The status of advancement in each of these topics is discussed.

INTRODUCTION

Grinding mill modeling has attracted the attention of a number of researchers ever since Bond published his method of energy calculation with an index called Bond work index (Bond 1961). In the 1970s, the population balance or selection and breakage function model was at the forefront of research (Fuerstenau et al. 1973). A number of advancements were made in the application of this model to ball mill grinding. However, the same models could not be extended to semi autogenous grinding. Simply carrying out pilot scale SAG grinding is a very tedious task. At this time, the discrete element method appeared on the scene. DEM helps in simulating a number of features of SAG mills and AG mills, not to mention balls mill as well. The principal advances made with DEM are discussed in this manuscript.

MILL POWER DRAFT

The discrete element method in its entirety was first published tin the civil engineering literature (Cundall and Strack 1979) from where it spread rapidly into many engineering disciplines including the mineral processing industry. In the 1970s the prevalent theme in the comminution literature was the advancement of Bond's formula (Bond 1961) for mill power prediction. The torque arm formula was the predominant theory (Hogg and Fuerstenau 1972; Guerrero and Arbiter 1960; Hlungwani et al. 2003). The torque needed to sustain the offset of the center of gravity of the charge from the mill center was being refined in many ways. The culmination of such efforts led to the more detailed empirical formula (Morell 1992) which included the toe and shoulder angle. The success of this model is due to its ability to define the charge boundaries. Yet all of these models were lacking in their ability to respond to change in lifter geometry or ball size distribution. This is where DEM made its mark, since it accounts for cascading charge, interpenetrating balls between layers in the cascade and also cataracting charge. Here the energy consumed in thousands of collisions of balls with other balls, mill walls and lifters is summed to arrive at mill power. Since the

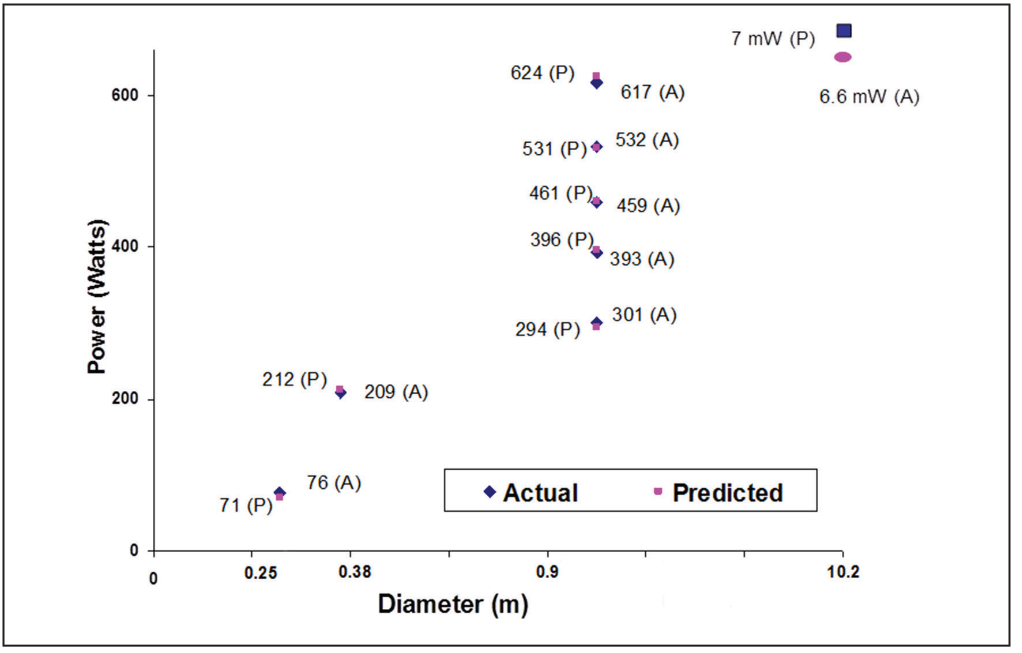

Figure 1. Comparison of actual power and predicted power draw using DEM for different mill diameters

geometry of mill internals and ball size distribution is taken into account explicitly, DEM computed power responded to variations in lifter design and ball size distribution.

The precursor to DEM is the computation of the trajectory (Powell 1991) of a single ball while it is in contact with the leading face of the lifter. The balance of forces due to gravity, centrifugal, sliding and rolling gave rise to accurate trajectories of a single ball. Even today this methodology is used in SAG mill lifter design. The single ball trajectory is a valuable tool for visualization of the parabolic trajectory of a ball and hence the strike zone on the toe of the mill (Sepulveda 2012). In fact, the best choice of lifter release angle can be determined using single ball trajectories. The best angle corresponds to the trajectory that can be maintained even as the lifter wears out (Royston 2007).

In DEM (Datta et al. 1999; Rajamani et al. 2003) the impact energy in each and every collision is summed for one or two full revolutions and is then converted to the mill power draft over the full mill length. While in 3D simulations the power can thus be computed directly, in 2D the power draft in a slice of the mill is computed and is then converted to power draft of the full mill length. The author has successfully computed SAG and ball mill power in hundreds of case studies besides verifying power prediction in laboratory size mills as shown in Figure 1 (Rajamani et al. 2000). The capability of 2D DEM for power predictions (Nierop et al. 2001) at as high 120% critical speed was done in a 55 cm mill. Given the uncertainties in power measurements 2D DEM is a viable tool. However, one could always claim the superiority of 3D DEM over 2D DEM for power predictions (Cleary 1998) which cannot be refuted by simple arguments. Further with 3D DEM a slice of SAG mill (Cleary 2001a) can be simulated to predict full mill power.

A further advance in DEM model would be to change the shape of particles from disk or sphere to non-circular and non-spherical shape. When advancing the DEM model from 2D disk to 2D non-spherical to 3D slice to 3D full (Cleary et al. 2003) the prediction of shoulder angle, toe angle, power and vortex center increasingly improved. Obviously, these findings point in the direction of 3D DEM for full scale charge motion plus breakage studies. A typical screenshot of 2D DEM charge motion is shown in Figure 2.

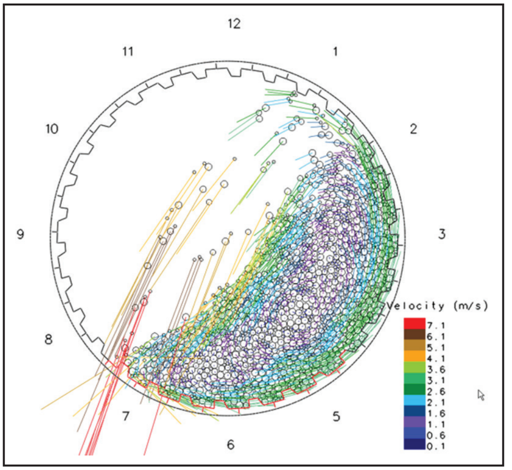

Figure 2. Typical screenshot of 2D DEM charge motion

The fundamental structure of DEM simulation rests primarily on the spring-dashpot contact model (Rajamani et al. 2000). There are spring constants, dashpot constants and friction parameters that must be set correctly. These parameters are difficult to determine experimentally and most researchers depend on the overall prediction of mill power as a goalpost for verification. Yet these parameters can affect normal and shear directional collisional energy on individual particles. In a careful photographic study (Chandramohan and Powell 2005) of binary collisions and particle bouncing on a plate, the shear mechanisms and sliding friction model imposed on the ubiquitous spring-dashpot model was found to be woefully inadequate. On the other hand a study of bouncing ball inside a mill shell (Dong and Moys 2002) found that spring-dashpot model is accurate enough as long as coefficient of friction and coefficient of restitution is correctly chosen. Regardless, the spring-dashpot model continues to be the working model in many DEM research groups.

LIFTER DESIGN

In the early 1960s, Art MacPherson investigated the effect of lifter spacing to height ratios and concluded that a ratio of 4:1 maximized grinding. Such designs were carried into Highland valley Copper, Copperton and other concentrators. While such designs lived up to the power draft expectations, at least one of them, Freeport's Grassberg operation reported severe damage to the SAG mill shell in the first few months after commission. DEM with its ability to animate the charge motion on a PC screen, as shown in Figure 2, comes to the rescue in such scenarios. It was found that the 14-inch tall lifters were causing direct ball strikes across the mill shell. This study would form the basis for future shell lifter designs. For the first time, the lift of charge, cascading, cataracting, packing and other aspects was viewed directly. In fact Minera Alumbrera mine (Sherman and Rajamani 1999) was the first mine, based on a DEM study incorporated 30 ° relief angle lifters, with considerable success. This study showed that the number of lifters needed for a mill of diameter D (in feet) was just D lifters instead of the "twice the diameter" rule that had been in existence. The key to lifter design was the fact that DEM was showing charge motion accurately. Hence, a number of researchers including the author (Cleary 2001a,b; Monama and Moys 2002; Hlungwani et al. 2003; Rajamani et al. 2000; Venugopal and Rajamani 2001) employed a variety of laboratory scale mills to verify DEM charge motion. At this time a spectacular advancement in filming the entire charge motion with X-ray and Gamma-ray was also developing (Kallon et al. 2011). After so much of verification, DEM has become a trusted tool for shell lifter design (Herbst and Nordell 2001; Rajamani et al. 1999) among liner manufacturers and plant process engineers.

LINER WEAR

Naturally, if DEM can track the influence of liner design it can also track liner wear. Liner wear life is an important to all concentrators. One could not have imagined modeling liner wear prior to DEM. The key information to wear modeling is in the heart of discrete element computations. They are the shear force, velocity and contact location on a lifter bar, shear energy at contact and normal energy at contact. Some use the well-known Archard's law (Powell et al. 2011; Herbst and Qiu 2007) and some use unique prescriptions of wear based on collision energies (Kalala et al. 2005 a, b). The sound concepts behind these wear models are reflected in the accurate wear profiles precited for a wave liner in a coal mill or block type lifters in copper ore grinding. In a carefully conducted study it was shown that the wear life of a 40 ft. SAG mill could be accurately modeled, and hence this study led to a unique design of a curve shaped lifter (Hart et al. 2006) with a lead angle of 28 degrees. It should be understood however the wear rate constant is unique to the ore slurry and metallurgy of the lifter bar.

Since steel loss is a significant contributor to cost of metal production, it would be obvious to account this loss on a quarterly basis. While DEM may be a predictive tool a practical measurement tool would be more preferable. To this end, today a system known as Mill mapper (Toor et al. 2011a) has the capability to map the entire interior of large mills. With this tool mill operations can now quantify the steel losses and take actions to reduce losses. It would be a matter of time before DEM is coupled with this measurement to take cost saving measures in plants.

PULP LIFTER FLUID FLOW

The next potential area of application of DEM is pulp lifter design. Plants are well aware of pooling issues that is detrimental to mill throughput. Furthermore, pulp lifters are inherently prone to carry over flow and flow back. Then there is the issue of radial pulp lifter versus curved pulp lifters; which one is suited for a particular SAG mill operation? Finally, the quantity of flow through grates into the pulp lifter is still lacking. Plants often increase open area of grates with the hope of increasing mill throughput. However, the back flow from the pulp lifter increases simultaneously thus nullifying the anticipated increase in throughput. DEM could be used to simulate slurry discharge profiles in the pulp lifter by modeling the slurry by a number of discrete slurry particles (Rajamani 2007). While valuable inferences in carry over flow can be drawn from such simulations, it lacks the viscous momentum transport model that is so essential to fluid flow. The obvious solution is to couple computational fluid dynamics (CFD) with DEM as has been done in the chemical engineering literature. The price to be paid for such rigor is computational complexity and so excessive computer time. The solution is smoothed particle hydrodynamics (SPH) algorithms. Just like DEM, SPH handles fluid flow via particles and hence is referred to as mesh less algorithm. These particles are given properties of viscosity and interaction with solid walls etc. In summary this is an ideal scheme to couple with DEM. Then the slurry flow through the porous ball charge (Cleary et al. 2006; Herbst and Nordell 2011) can be modeled and then the slurry can be followed during its transit through the grate and then into the pulp lifter until it exits via the discharger. Thus the momentum transfer from the ball charge to the slurry enables the computation of slurry flow in the mill discharge.

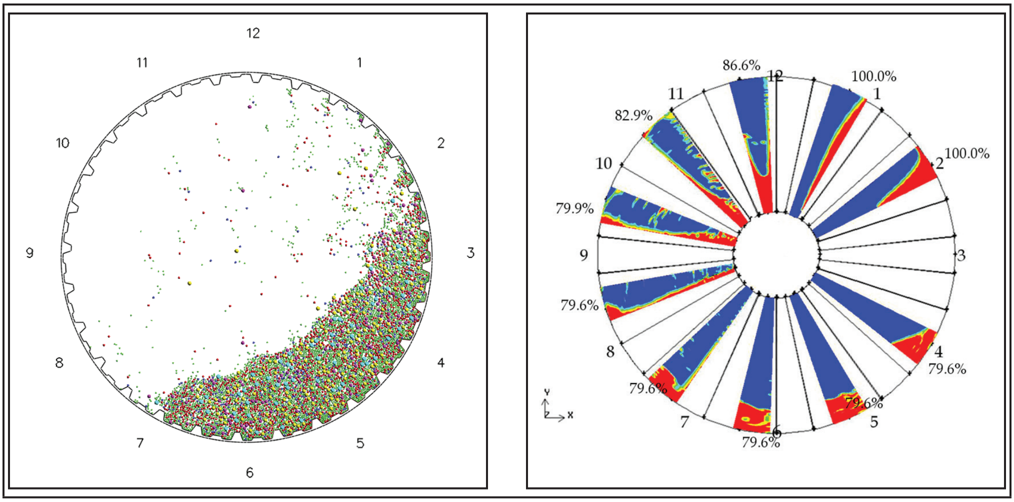

Figure 3. DEM charge profile and CFD prediction of free surface profiles as the mill discharge, Los Bronces SAG mill (diameter 8.26 m and length 4.19 m) (Rajamani et al. 2011)

Another alternative to DEM with SPH coupling is direct CFD simulation of a single blade in the pulp lifter. The charge profile in the mill interior is simulated with DEM. The profile of charge in the neighborhood of 7, 6, and 5 pm regions indicates the approximate level of slurry in the blade of a pulp lifter. It may be argued that slurry level above mill charge may vary considerably. Nevertheless, this approximation is quite reasonable. Then the full CFD simulation of the blade (Rajamani et al. 2011) is carried out; summing over all the blades gives the overall discharge through the mill. This effort has been very successful when compared with published information on six SAG mills; one example is shown in Figure 3. In summary, advances in slurry flow via DEM is continuing.

PARTICLE BREAKAGE IN MILLS

DEM can be readily employed to model breakage since the force of impact and energy of impact are the back bone of DEM algorithm. However, the intractable nature of ore particle breakage itself is a severe challenge. A fundamental theory of ore particle breakage under different impact loads is nonexistent today. Even if one existed it would not be sufficient since particles in a mill break under multiple point stresses. Nevertheless, describing the evolution of ore particles size distribution is the frontier of DEM applications to grinding mills.

The approach to describing particle fracture varies in its complexity. The prevailing charge can be assumed as rigid particles and the spectrum of impact energy in all of the collisions can be taken as the driving force for breakage of particles. It is well known in the modeling of breakage, breakage rate or selection function and breakage distribution function categorize the breakage into a set of size classes. Therefore, the formalism is to convert the impact energy spectrum into selection and breakage functions. Such formalism leads to a model that is akin to the selection and breakage model known popularly as population balance model of grinding. In such an approach the time evolution of size distribution was successfully predicted (Datta and Rajamani 2002). The same

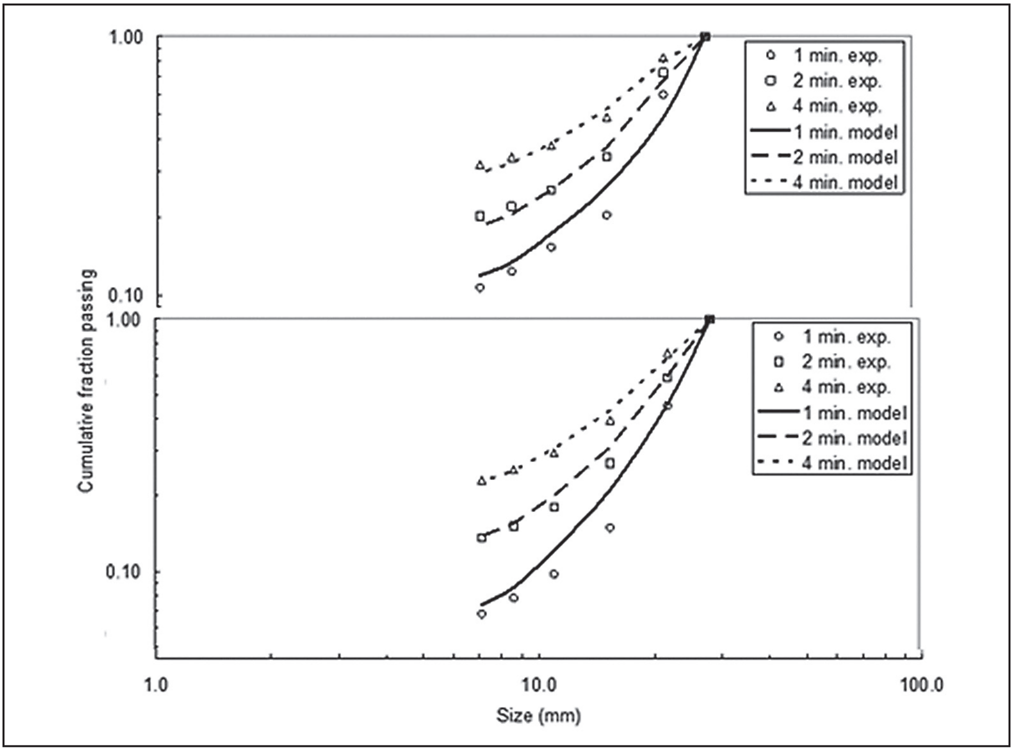

Figure 4. DEM-based prediction of ore size distribution in a 90 × 15 cm ball mill operated at 26% and 21% loading

methodology was extended to a pilot size mill and the predictions are shown in Figure 4 (Tuzcu and Rajamani 2011).

Yet another method to arrive at broken particle size distribution is to characterize breakage of particles in a drop weight apparatus. The results can be linked in a suitable functional form between broken particle size and applied energy. Then in DEM (Morrison and Cleary 2004; Cleary and Morrison 2011) the computed normal and shear energy of collision on individual particles can be used in conjunction with the derived functional form to determine progeny size distribution. A key expression for converting impact energy into probability of a particles survival (Vogel and Peukert 2004) is shown in Equation 1. A more convenient form of the above equation for DEM is shown in Equation 2.

$$s = 1 - \exp\left\{-f_{mat} * x * k\left(w_{m,kin} - w_{m,\min}\right)\right\} \quad \textbf{(1)}$$

$$s = 1 - \exp\left\{-\acute{b}\sum_i\left(E_i - E_0\right)\right\} \quad \textbf{(2)}$$

where, s is the breakage portion, $w_{m,kin}$ is the single impact energy, $w_{m,min}$ is the no damage energy, x is the particle diameter, k is the number of impacts and f_{mat} is the material parameter, b is the material parameter, E_i is the ith collision energy and E_0 is the no damage energy.

These expressions together with a model of progeny distribution upon breakage is a convenient frame work in DEM since successive impacts can be recorded and so the damage on a single particle can be accumulated and be broken at the appropriate time (Morrison et al. 2011). Thus, it is possible to model both attrition and impact breakage in SAG/AG mills. Nevertheless, modeling the breakage due to accumulating damage and multi point contact breakage is a complex task. More detailed models quadruple the time investment in DEM computing.

A much rigorous approach is to consider the continuous spectrum of collision energies from which a fraction of the impact energy is absorbed by the particle. Once again, a continuous breakage function is defined for particles broken in this manner. Next a particle surface model irrespective of the collision energy is defined. Then the continuous impact energy distribution (computed with DEM) can be allocated to body and surface breakage and the respective progeny distributions are modeled to include in the appearance term of the mass balance. In essence such a micro scale model owing the scale employed is extremely complex in its mathematical formulation, yet it carries lot more detail into the computations (Tavares and Carvalho 2010; Carvalho and Tavares 2012). These models continue to improve as computational power advances every year.

Finally, if one wants to go into as much detail as following each and every particle undergoing breakage in the mill then the discrete grain breakage (Herbst and Potopov 2004; Herbst and Nordell 2001) model belongs in this category. DEM is used in the complete description of the grinding media. At any instant of time in DEM the particles are allowed to break into progeny particles according to the discrete grain model. Next, the progeny particles are included in the subsequent time steps in DEM. Although promising, this numerical technique requires enormous computational resources such as a CRAY-super computer. In addition, because each ore particle is followed in the algorithm a discrete grain liberation (Herbst and Qiu 2007) model too can be imbedded. Finally high computational resources enable one to include SPH for the simulations formulation of slurry flow. In summary there are a few different formulations of particle breakage in DEM. All of them are advancing year by year. As of today, there is no single formulation that can encompass all of the complexity of ore breakage in grinding mills.

MEASUREMENTS TO VALIDATE DEM

While DEM is progressing, there are two experimental techniques that closely mimic DEM. The first one is positron emission particle tracking (PEPT) (Parker et al. 1997) which is a technique for tracking granular particles inside the mill shell. The basis of PEPT is the labeling of a particle with a radio nuclide which decays via the emission of a positron, the antiparticle of an electron. While the positron decays two simultaneous gamma rays are emitted defining the line of emission. The PET camera surrounding the shell detects many thousands of lines of emission which is then processed to locate the coordinates of the particle precisely within the shell. PEPT is now (Kallon et al. 2011) capable of quantifying circulation rate of charge particles, the profile of the charge and the velocity of individual particles. This technology is evolving and perhaps someday it will be a tool for diagnostics on a plant scale.

Another technology that can take the benefit of DEM simulation is the mill-mapper: a 3D laser scanner mounted inside the mill collects the 3D volume map of the mill interior space. The point cloud is then analyzed to provide mill liner thickness, liner shape, liner weight, and mill volume. Since it provides liner height and cross sectional geometry of shell lifters, DEM can be combined with this data to assess the effective breakage regime during the life time of a lifter set (Toor et al. 2011b).

DEM COMPUTATIONS

Thus the DEM has grown to accommodate a number of aspects of tumbling mill operation. Further, the scale of simulation is at the level of plant scale mills in three dimensions. Obviously, such computations require enormous computing resources and computational time. Notwithstanding weeks and weeks of compute time, one employs multi-node processors, CRAY computer and parallel computing algorithms and so on. A recent advancement in this respect is GPU computing (Rajamani et al. 2011). Here the three-dimensional computing is done on the video card of the desktop PC. Commodity graphic cards deploy hundreds of processors in the computation. The algorithms are written in OpenCL or CUDA language, which deploy the calculations on hundreds of processors automatically. In fact a speed up factor of fifty was established for the Nvidia Geforce 580 graphics card over single CPU. The simulation of a SAG mill with 250,000 spheres was completed in 8 hours and the simulation of ball mill with one million spheres was completed in 27 hours. A typical snap shot of DEM-GPU combination for a plant ball mill is shown in Figure 5. GPU is a strong candidate for the future of DEM simulations since the card hardware seems to advance every six months.

CONCLUSION

DEM has advanced in the grinding mill field since its introduction in 1991. The prediction of mill power draft, charge motion and lifter design has matured to an advanced level. Particle breakage in mills is advancing rapidly despite the difficulty inherent in this phenomenon. DEM computations stretch the limit of available resources, yet progress has been made in this front also. In the near future DEM will become the standard tool for design of mills, if not already.

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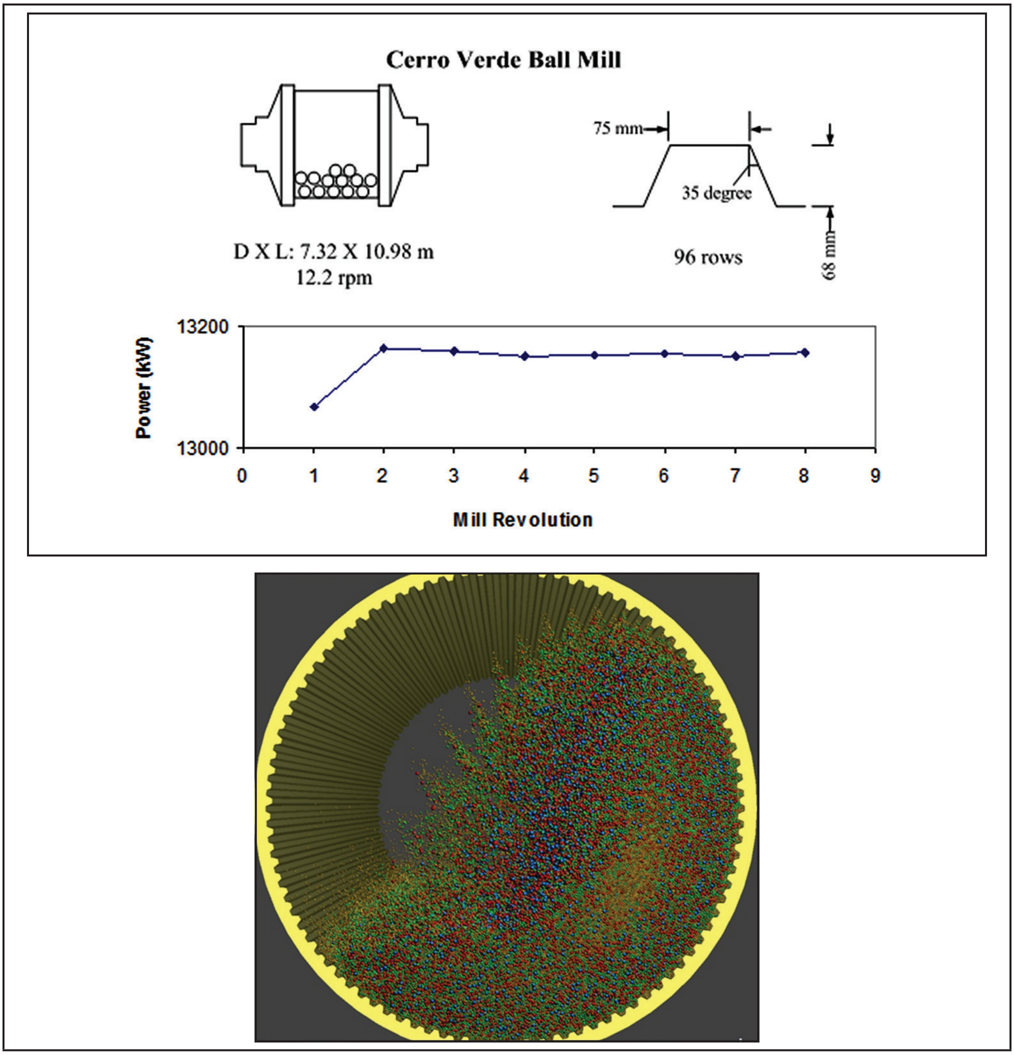

Figure 5. Simulated power draw of the Cerro Verde ball mill with 3D GPU snapshot

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